# A NEW AND EASY METHOD OF FINDING THE LATITUDE ON SEA OR LAND,

HAVING

Two ALTITUDES of the SUN

AND

The TIME ELAPSED between the two OBSERVATIONS,

Measured by a WATCH or other TIME-KEEPER.

By SAMUEL DUNN,
Teacher of Mathematics.

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IN a Treatise on *The Theory and Practice of the Longitude at Sea*, lately published, at Page 123, I have given a Specimen of a general Method, whereby the Latitude may be determined, by having *any* two Altitudes of the Sun, the Elapsed Time between the two Observations correctly measured by a Watch or any other Time-keeper, the Sun's Declination and the Latitude somewhat near the Truth, as it is commonly known by the Reckoning of a Ship at Sea; the Calculation being easily made by the common logarithmic Tables without any consideration of the Properties of Spherical Triangles. But as this Method is there delivered in a concise manner, I have drawn up the following Account of the Principles on which it depends, to obviate any Difficulties that may arise in the Application of it either by Sea or Land.

When an Observation is intended to be made by this Method, the Quadrant or Sextant should be adjusted as correctly as possible, and the Watch or other Time-keeper by which the Elapsed Time is to be measured should be brought to such an equable Rate of going, that it may be depended on for measuring the Interval of Solar Time, very near the Truth. Nor should either of the Altitudes be taken when the Sun is very near the Meridian, especially when the Place of Observation is remote from the Equinoctial, because then the Sun's Altitude changes slowly; but at any other times of the Day the two Altitudes may be taken, allowing as usual for the Dip of Horizon, Refraction, and Semidiameter when the lower Limb is brought to sweep the Horizon of the Sea.

The Elapsed Time measured between the two Observations may be about an Hour, or less will be sufficient in Cases where it is measured accurately and the two Altitudes are taken with but very little Error; otherwise, a longer Interval of Time may be more advantageous for an accurate determination of the Latitude. But when such Interval of Time is long and the Sun's Declination changes from one to two or three

three Minutes of a Degree, between the Obſervations, the Declination and Polar-diſtance found therefrom, ſhould be taken out for both of the times of Obſervation as correctly as poſſible.

Under theſe Circumſtances, it will be ſcarce neceſſary to know whether the two Altitudes are taken one before and the other after Noon, or both before or both after Noon, if they are made on the ſame Day; becauſe, the two Hour Angles or Diſtances of the Sun from Noon at the times of Obſervation, being computed (in uſual Caſes) when the Sum of them agrees neareſt with the meaſured Elapſed Time, the Obſervations will have been made one of them before and the other after Noon; but otherwiſe, when the Difference of them agrees neareſt with the meaſured Elapſed Time they will have been made both before or both after Noon.

It is farther to be noted, that during the Elapſed Time, the Ship may have ſo altered her Place by the Courſe and Diſtance ſailed, that when the ſecond of the two Altitudes is taken, it will diſagree with an Altitude that would be taken that ſame Inſtant at the Place where the firſt Obſervation was made, from which Place the Ship is removed. This muſt be allowed for, thus. 1ſt. If the Ship hath ſailed directly toward the Sun, the Miles ſhe hath ſailed in the Interval are the Minutes of a Degree to be ſubtracted from the Altitude at the ſecond Place to reduce it to the firſt. 2d. If directly from the Sun, they are to be added. 3d. If directly at Right Angles to either of the former Courſes, nothing is to be added nor ſubtracted. 4th. If on any other Courſe, the Subtraction or Addition is to be made, of as many Minutes as the Courſe and Diſtance in the Elapſed Time hath either carried the Ship toward or from the Sun, from the firſt to the ſecond Obſervation.

Theſe things being attended to concerning the Data or things given, the Obſerver will be provided with five things; namely, the two Altitudes each in Degrees and Minutes, the Elapſed Time turned into Degrees and Minutes by allowing 15 Degrees to an Hour of Time, the Sun's Declination and conſequently his Polar-diſtance, the Latitude by the Ship's Reckoning imperfectly known, to which he is to add a ſixth Term or thing given, namely another Latitude greater or leſs than that of the Reckoning, at Direction, as he may judge the Latitude by the Reckoning to be either erroneous or uncertain

certain; and from theſe, the true Latitude may be found by the following Method of Computation.

With the Co-latitude by the Ship's Reckoning, the Polar-diſtance and each of the two Co-altitudes compute the two Hour Angles from Noon for the two times of Obſervation, and their Sum or Difference (that of the two agreeing neareſt with the Elapſed Time) will be the firſt Proof that the Latitude by the Reckoning is either true or erroneous; for if it be true, it will be the ſame as the Elapſed Time; otherwiſe it will be more or leſs, and another operation is to be made thus.

With the Co-latitude newly aſſumed (either greater or leſs than that by the Reckoning) and the other things before mentioned, get the Elapſed Time in the ſame manner as before mentioned, and take the Difference between it and the Elapſed Time found by the Co-latitude of the Reckoning.

When both of theſe Elapſed Times are found, it will plainly appear, which of the two Latitudes they have been computed from, is neareſt to the true Latitude, if both have been too great or two little; and if one Elapſed Time be greater, but the other leſs than that meaſured by the Watch, the true Latitude is between thoſe two Latitudes that have been uſed. Therefore, in this Caſe,

Subtract one computed Elapſed Time from the other, the Remainder is a firſt Term in Direct Proportion; a ſecond Term is the Difference of the two aſſumed Latitudes; and a third Term is the Difference between the meaſured Elapſed Time and either of the two computed ones; therefore, multiplying and dividing, the fourth Term will be the Miles or Minutes of Latitude to be added to or ſubtracted from that Latitude which had it's correſpondent Difference compared with the true Elapſed Time meaſured by the Watch. In other words thus:

As the Difference of the Elapſed Times computed from the aſſumed Latitudes; is to the Difference of thoſe Latitudes: ſo is the Difference between the true Elapſed Time and either of the computed Elapſed Times; to a Number of Minutes, which added to or ſubtracted from the correſpondent aſſumed Latitude, gives the true Latitude required, when the Latitudes are aſſumed near enough for the Truth.

In

In the forementioned Treatiſe on the Longitude at Sea, Page 128, I have given a Specimen of the manner in which ſuch Calculations may be made, by a Copper Plate Engraving, which is alſo annexed to this Tract, in order to the better explanation of the Principles on which this Method is founded. In this Example, the Latitude by the Ship's Reckoning is ſuppoſed 46d. 20m. N. the two Altitudes 46d. 55m. and 54d. 7m. the Elapſed Time 1h. 25m. or 21d. 15m. and the Declination 12d. 17m. N. and as the Latitude by the Reckoning is not ſuppoſed correct, another Latitude as 46d. 36m. is aſſumed, whereby to proceed through the Calculation and get the true Latitude required.

The Declination is ſubtracted from 90d. becauſe it and the Latitude are alike, otherwiſe it would have been added to 90d. to get the Polar-diſtance. From the half ſum of the Co-latitude, the Polar-diſtance and Co-altitude; the Co-altitude is ſubtracted to get a Remainder. The Co-ſecant of the Co-latitude leſs 10. in the Index; the like for the Polar-diſtance, the Sine of the half Sum and the Sine of the Remainder, are added together; half their Sum is found in Sines; and twice the correſpondent Coſine Arch is the Hour Angle from Noon. Laſtly, the Difference of the Hour Angles computed for the ſame Latitude and both Altitudes, agreeing nearer than their Sum with the Elapſed Time meaſured by the Watch, ſhews the Obſervations were made when the Sun was on the ſame Side of the Meridian. The firſt computed Elapſed Time is 20d. 34m. for the Latitude 46d. 20m.; the ſecond is 21d. 42m. for the Latitude 46d. 36m.; but the true Elapſed Time being 21d. 15m. the true Latitude is between the two aſſumed ones; therefore, the proportional Part is additive to the leaſt Latitude, as directed in that Specimen or Formula; this may be called Caſe the firſt. The ſecond Caſe may be when both of the computed Elapſed Times are either greater or leſs than the meaſured Elapſed Time.

For Inſtance, ſuppoſe the Elapſed Time meaſured by the Watch had been 21d. 50m. then it is evident, the Latitude would have been more than 46d. 36m. Therefore, from 21d. 50m. ſubtract 21d. 42m. remains 8m. And, as 68 to 16m. ſo 8 to 2m. nearly, which added to 46d. 36m. gives 46d. 38m. the Latitude ſought.

Otherwiſe

Otherwiſe, if the Elapſed Time meaſured by the Watch had been 20d. 30m. then it is evident the Latitude would have been leſs than 46d. 20m. Therefore, from 20d. 34m. ſubtract 20d. 30m. remains 4m. And, as 68 to 16m. ſo 4 to 1m. nearly, which taken from 46d. 20m. gives 46d. 19m. the Latitude ſought.

This Method was publiſhed by me the 1ſt. of this preſent Month, after having been firſt entered at Stationers Hall, in the forementioned Treatiſe, ſeventeen Days before the Publication. Since the Publication of that Book, I have taught it to ſeveral Perſons, who have been pleaſed to give it no little Encomiums on Account of the Eaſe and Certainty wherewith it may be practiſed. One Gentleman, whom I have leave to refer to, aſſures me that the 17th. of this Month, he took the Latitude by it, about half a Mile Weſt of St. Paul's London, without erring ten Seconds of a Degree.

In aſſuming the two Latitudes, when the three Sides of the Spherical Triangle which begin the Work have not been in an extreme manner different from one another, I have found the true Latitude by the firſt Aſſumptions, within a Minute of a Degree, although they have been aſſumed many Degrees different from the Truth. But in ſome particular Caſes, when one of thoſe Numbers happen to be very ſmall in Compariſon with the others, the Convergency toward the Truth will be more ſlow. However, this is eaſily remedied by replacing the newly-found Latitude inſtead of the former, and repeating the Operation, until there is but a ſmall Difference between the meaſured and the computed Elapſed Time.

In this Example, but five Places of Logarithms are uſed, but if more Places of Logarithms are applyed, the Reſult will be more correct; ſuch will ſeldom be wanted in uſual Caſes. In the preſent Example (which is not a very favourable one) the mean Difference of the Elapſed Times, ariſing from each Minute of Latitude aſſumed, is between 4 and 5 Minutes of a Degree; and by proportioning for an Interval of 20 Minutes of Latitude, the greateſt Error would ſcarce amount to one Mile, and this may be eaſily corrected by a new Aſſumption. Thus,

Lat. 46d.

| Latitudes. | El. Times. | Latitudes. | El. Times. |
|---|---|---|---|
| 46d. 20m. | 20d. 33m. | 46d. 30m. | 21d. 12m. |
| 46d. 21m. | 20d. 36m. | 46d. 31m. | 21d. 17m. |
| 46d. 22m. | 20d. 40m. | 46d. 32m. | 21d. 21m. |
| 46d. 23m. | 20d. 44m. | 46d. 33m. | 21d. 25m. |
| 46d. 24m. | 20d. 48m. | 46d. 34m. | 21d. 30m. |
| 46d. 25m. | 20d. 52m. | 46d. 35m. | 21d. 36m. |
| 46d. 26m. | 20d. 56m. | 46d. 36m. | 21d. 41m. |
| 46d. 27m. | 21d. 0m. | 46d. 37m. | 21d. 46m. |
| 46d. 28m. | 21d. 4m. | 46d. 38m. | 21d. 51m. |
| 46d. 29m. | 21d. 8m. | 46d. 39m. | 21d. 57m. |

As the Dip of Horizon (or Allowance for the Height of the Eye above the Surface of the Sea) is to be known in taking Altitudes at Sea, I ſhall here inſert the Table in Page 146 of the *Theory and Practice of the Longitude at Sea*, which was computed by me from the lateſt Diſcoveries concerning the Earth's Diameter.

| Height | Dip | Height | Dip |
|---|---|---|---|
| 1 foot, | 1 min. | 32 feet, | 6 min. |
| 3½ feet, | 2 min. | 43 feet, | 7 min. |
| 8 feet, | 3 min. | 57 feet, | 8 min. |
| 14¼ feet, | 4 min. | 72 feet, | 9 min. |
| 22 feet, | 5 min. | 89 feet, | 10 min. |

But for a *Table of Refraction*, refer to the general one in that Treatiſe, and the Method of applying it at all the principal Parts of the Earth and Sea, for any Refraction at the Horizon.

For nicer Corrections in this Subject, I refer to *Practical Navigation*, from Page 100 to 118; where are ſeveral things relative to this Doctrine, not treated of by any other Writer.

Maiden Lane, Covent Garden,
July 21ſt. 1778.

S. DUNN.

## *Of the Practice and Utility of this Method, in Aſtronomy, Geography and Navigation.*

In the foregoing Pages it hath been ſhewn what are the general Principles on which this *New Method of finding the Latitude* depends, and how the Calculations may be made by the common logarithmic Tables; to which it ſeemed not improper to add the circumſtances concerning the order of Time and manner in which it was firſt publiſhed. Every Perſon to whom I have perſonally communicated this Method, has admitted it as being eaſy in Computation, and that the logarithmic Sines, and others to be taken from the Tables, have peculiar Advantages; and that theſe are not all belonging to the Problem, will appear from the Principles on which it's Solution is founded.

The principal Data are, the two Altitudes, the Polar-diſtance, and the Elapſed-time; theſe ſhould be known as correctly as poſſible, and in any Method of Solution, if there be a Deficiency or Imperfection in either of theſe, the Latitude determined therefrom will become erroneous. To which may be added, that whatever Method of Solution be applied on the Hypotheſis of a ſpherical Earth, the Inequalities ariſing on a ſpheroidal one will affect the Calculation, and therefore for an accurate Determination, that Allowance ſhould be made, otherwiſe it will be needleſs to apply the ſmaller Subdiviſions of ſpherical Triangles.

It is likewiſe to be obſerved, that although the two Altitudes, the Polar diſtance and the Elapſed-time, be accurately known, the Latitude by the Ship's Reckoning may be greater or leſs than the true Latitude by ſuch a Number of Degrees or Minutes, that the Elapſed-time cannot be computed from it and the other Data. This at firſt Sight ſeems to be an Imperfection, but it is a great Advantage in this Method of Solution. For Inſtance, in the preſent Example, where the Latitude by the Ship's Reckoning is ſuppoſed 46d. 20m.; had this been ſuppoſed 47d. 20m. the Co-latitude would have been 42d. 40m. the Polar-diſtance being 78d. 43m. and the ſecond Co-altitude 35d. 53m. from which, it is evident the Co-latitude 42d. 40m. would have been aſſumed too little, or

what amounts to the ſame thing, the Latitude 47d. 20m. too great for the Truth; becauſe the Co-latitude and Co-altitude together, would have been leſs than the Polar-diſtance. and the three ſuppoſed Data would have made no Triangle, two of the three together being leſs than the third. This would have ſhewn, that the firſt aſſumed Latitude ſhould have been leſs than 47d. 20m. by ſuch a Difference that the Sum of any two of the three given Terms muſt have been greater than the other of thoſe three Terms.

For the ſame Reaſon, the ſecond aſſumed Latitude, muſt have been leſs than 47d. 20m. otherwiſe it would not have formed the Sides of a Triangle. Otherwiſe, when the three Terms are ſuch, that two of them together do not make the third, the half Sum of the four logarithms, will have it's Index more than 9. and conſequently, no Co-ſine Arch can be taken out, anſwering to it; and this alſo would imply or diſcover the Impoſſibility of ſuch a Latitude agreeing with the two Altitudes, the Polar-diſtance and the Elapſed-time.

Hence an eaſy Method ariſeth of making the Aſſumptions at once, and in ſuch a manner that they will need no Correction in a great Variety of Caſes. For, in any one of the Computations, the Co-latitude muſt be greater than the Difference between the Polar-diſtance and Co-altitude. If it be equal thereto it makes no Triangle; if it be greater it doth, and the Computation will bring out the Hour-angle. This is a great advantage peculiar to the Method, when the Triangle happens to be very oblique; but when it is otherwiſe, or there happens to be a nearer Approach to Equality amongſt the Sides compared with one another, no ſuch Impoſſibility will happen, therefore, in ſuch Caſes, the Aſſumption, may be made at diſcretion, and the reſult will come out by the uſual Theorem for that purpoſe.

Another Advantage attending this Method of Computation, is that of being able to take out the Complements-arithmetical, the Sines and Coſines, for ſeveral Data near the ſame Pages. Thus for Inſtance, in the preſent Example; the Complement-arithmetical of the Polar-diſtance is taken out at once, although it be written four times in the Calculation. The ſame Co.ar. for the Co-latitude is taken out at once, although it be written twice in the Calculation. Alſo, by Reaſon of the nearneſs of the Aſſumptions to each other, which

(which in uſual Caſes may not exceed 30 Minutes of a Degree) the Sines of the Half-ſums, may be taken out near the ſame logarithmic Pages; the like for the Sines of Remainders and the Coſine Arches. If theſe Conveniences be attended to, a great Part of the Time that would otherwiſe be taken in turning from one Page to another for each Number ſeparately, will be avoided, and the whole Calculation be not only eaſily performed, but an Error (if any) in either of the Parts of the Computation the more eaſily diſcovered, by comparing the Angles reſpectively with each other, when they are found by Calculation.

Hence alſo it appears that there will be ſcarce any occaſion for other logarithmic Tables than are in almoſt every Navigator's hand, and in almoſt every Book of Navigation; namely, the logarithmic Sines, Tangents and Secants, and a few Places of thoſe Logarithms may be ſufficient in uſual Caſes. Nor will there be wanting any other Theorem for making the computation by, than that which is ſo well known for computing the Azimuth Angle. But in the Application of it to this Problem, for computing the Hour-angle from Noon, the firſt Term will always be the Co-latitude, and then the general Proceſs will be performed thus:

Add together the Co-latitude, the Polar-diſtance and Co-altitude, and from their Half-ſum ſubtract the Co-altitude, to get a Remainder. Then add together the Co.ar. of the Colatitude, the Co.ar. of the Polar-diſtance, the Sine of the Half-ſum, and the Sine of the Remainder; Half the Sum of theſe four Logarithms being found in Sines, twice the correſpondent Coſine Arch, is the Hour-angle from Noon ſought.

If the Declination and Latitude are alike, the Declination ſubtracted from 90 Degrees, but otherwiſe added to 90 gives the Polar-diſtance. The Logarithms when Degrees are more than 90d. are taken for the Remainders to 180d. The Co.ar. is Log. coſecant with it's Index leſſened by 10.

There is another Advantage attending the Calculation by this Method, concerning the order in which the computation may be made; for, although the Form ſhewn by the copper-plate Engraving be no way difficult, others may be uſed; amongſt which, the placing of the reſpective Co-latitudes, Polar-diſtances, Co-altitudes, ſums, Half-ſums and Remain-

ders under each other, and then the reſpective Co.ars, Sines, ſum of Logarithms, Half-ſums and correſpondent Coſine Arches, will be attended with no more Difficulty, and at the ſame Time are more ready for a Compariſon with each other. If this form be uſed, the preſent Example will be computed thus.

| | | | |
|---|---|---|---|
| 43.40 | 43.40 | 43.24 | 43.24 |
| 78.43 | 78.43 | 78.43 | 78.43 |
| 43. 5 | 35.53 | 43. 5 | 35.53 |
| 165.28 | 158.16 | 165.12 | 158. 0 |
| 82.44 | 79. 8 | 82.36 | 79. 0 |
| 39.39 | 43.15 | 39.31 | 43. 7 |
| 0.16086 | 0.16086 | 0.16299 | 0.16299 |
| 0.00848 | 0.00848 | 0.00848 | 0.00848 |
| 9.99650 | 9.99214 | 9.99637 | 9.99195 |
| 9.80489 | 9.83581 | 9.80366 | 6.83473 |
| 19.97073 | 19.99729 | 19.97150 | 19.99815 |
| 9.98536 | 9.99864 | 9.98575 | 9.99907 |
| 14.47 | 4.39 | 14.36 | 3.45 |
| 29.34 | 9. 9 | 29.12 | 7.30 |
| 9. 0 | | 7.30 | |
| 20.34 Diff. | | 21.42 Diff. | |

This ſeems to be the moſt natural and eaſy Form the Calculation admits of, and gives a View of the ſeveral Numbers repeated, and at the ſame time of thoſe which are nearly equal, by Reaſon of the nearly aſſumed Latitudes. The other particulars are deſcribed in page 5 of this Tract. Thus much for the Method of making the Calculation. It comes next to be conſidered what Uſe this Method may be of in Geography and Navigation.

However perfect the Latitudes of Places may be ſuppoſed to be in the beſt Maps and Charts, there is ſufficient Reaſon to conclude that many of them want no little Correction, and

as

as Opportunity for this, by Meridian Altitudes, is not always favourable when it is wanted, this is one Method whereby that Deficiency may be ſupplied, when the Obſerver is provided with a good Altitude Inſtrument and alſo a Time-keeper ſufficiently correct for meaſuring the Elapſed-time.

Therefore, this Method may be applied in ſettling the Latitudes of Places in the interior Parts of Countries, alſo of Places near the Sea Coaſts, and will be particularly applicable for that Purpoſe at Iſlands which are remote from other Lands, and but ſeldom viſited by Navigators. In this Senſe it may be conſidered as very aſſiſting for the Reformation and Improvement of Geography.

In the Lunar Method of finding the Longitude at Sea, this Problem may be uſeful for determining the Time of the Day at the Ship, becauſe without knowing the Latitude, the Hour Angle from Noon for the Place of Obſervation cannot be known.

It may alſo be uſeful in obſerving the Azimuth at Sea; becauſe, when the Latitude is found, it will be by the Property of a Spherical Triangle; As the Sine of the Co-altitude, is to the Sine of the Hour-angle from Noon; ſo is the Sine of the Polar-diſtance, to the Sine of the true Azimuth. And having the true Azimuth, it being compared with the magnetic Azimuth, gives the Variation of the Compaſs.

Near the Equinoctial, the Elapſed-time meaſurable between two Obſervations, may frequently be ſeveral Hours for different Parts of the ſame Day; and in higher Latitudes, during the Summer Seaſon, a longer Interval may be meaſured, without any Regard to particular times of the Day, excepting when the Sun is very near the Meridian; and certainly, this is far leſs diſadvantageous than the Methods which have been lately propoſed as practicable, by the Sun's Limbs paſſing an horizontal Line and meaſuring that Elapſed-time.

The Obſervations by this Method, may be made when the Sun is near the Prime-vertical or Eaſt and Weſt Azimuth Circle; where they will be leaſt affected by the Earth's ſpheroidal Figure, and at the ſame time the Altitude will increaſe or decreaſe faſteſt in the ſame time.

When the Sun's Declination has altered between the times of Obſervation, the Polar-diſtance taken out correctly for each of them, and the Calculation made in the uſual manner, determines

termines the Elapſed-Time according to the aſſumed Latitudes, as effectually as if there had been no Change of Declination in that Interval. Therefore, any Interval may be choſen, provided it be truly meaſured.

One, two or more Setts of Obſervations may be made by this Method near the ſame Place, and being compared with each other, the Medium may be more accurate than one Sett only.

Other Inſtances of the Uſe of this Method might be produced, but theſe may be ſufficient for a Specimen of it's general Utility. Alſo, by it the Latitude may be determined when a Ship wants to keep near a certain Parallel and make Land whoſe Latitude is known, which alone is ſufficient to recommend it to be carefully practiſed at Sea. However, it may not be unneceſſary to add one Example ſuch as the following of a ſhort Interval.

1778 June 23d. at 8h. 55m. 20s. per Watch I took the Sun's Altitude 45d. 26m. and at 9h. 30m. 30s. per Watch I took his Altitude 50d. 20m. ſo the Elapſed-time was 0h. 35m. 10s. or 8d. 47m. 30s. And becauſe the Refraction at each of theſe Altitudes was ſmall; alſo, there might be a ſmall Error in the Altitudes and another ſmall Error in eſtimating the Elapſed-time, I ſhall take the Co-altitudes as they were obſerved, and aſſuming the Latitudes 50d. and 52d. the Calculation will be thus.

| | | | |
|---|---|---|---|
| 40. 0 | 40. 0 | 38. 0 | 38. 0 |
| 66.33 | 66.33 | 66.33 | 66.33 |
| 44.34 | 39.40 | 44.34 | 39.40 |
| 151. 7 | 146.13 | 149. 7 | 144.13 |
| 75.33 | 73. 6 | 74.33 | 72. 6 |
| 30.59 | 33.26 | 29.59 | 32.26 |
| .19193 | .19193 | .21066 | .21066 |
| .03744 | .03744 | .03744 | .03744 |
| 9.98604 | 9.98083 | 9.98401 | 9.97845 |
| 9.71163 | 9.74112 | 9.69875 | 9.72942 |
| 19.92704 | 19.95132 | 19.93086 | 19.95597 |

9.96.

| | | | |
|---|---|---|---|
| 9.96352 | 9.97566 | 9.96543 | 9.97798 |
| 23. 9½ | 19. 0 | 22.33½ | 18. 5½ |
| 46.19 | 38. 0 | 45. 7 | 36.11 |
| 38. 0 | | 36.11 | |
| 8.19 Diff. | | 8.56 Diff. | |

Hence the Difference of the Elapſed-times ſhews the Obſervations were made in the Morning, becauſe they agree ſo much nearer than their Sum would with 8d. 47m. 30s. the meaſured Elapſed-time, and the Altitudes were increaſing. Then, as 37 the Difference of the Differences, to 120m. the Difference of the Latitudes; ſo is the Difference between 8d. 47½m. and 8d. 19m. (28½m.) to 92½m. or 1d. 32½m. which added to 50d. gives 51d. 32½m. the Latitude North, which is but 2½ Minutes more than it would have been found by the beſt Meridian Altitudes of the Sun or Stars; although no extraordinary Care was taken in making the Obſervations, and the Elapſed-time was but a little more than half an Hour, nor were any other Tables uſed but the common logarithmic ones. It is probable that a Correction of ſeveral ſmall Errors in the Data might have given the Latitude nearer the Truth, and it is certain that a longer Interval of Elapſed time would have given a more perfect Anſwer, but ſuch an one as this may be often of Uſe both on Land and at Sea. In this Example, the Sun was not far from the Prime-vertical or Eaſt and Weſt Azimuth Circle, in other Caſes it might have been queſtioned, how much the Latitude found had been affected by the Earth's Figure, the Corrections belonging to which have been before mentioned.

From this Example, the former, and others, ſuch as happen in uſual Caſes; it appears, that when the two Latitudes have been aſſumed ſomewhat near the Truth, the proportional Part, whether additive or ſubductive, will be very near the Truth; and therefore, the nearer the Latitudes are aſſumed, and the longer the Elapſed-time is (both of the Altitudes being taken when the Sun is not very near the Meridian, nor under the greateſt Affection by the ſpheroidal Horizon, nor Error by Refraction) the more accurately will the Latitude

Latitude found by this Method agree with real Obſervations.

Hence ariſeth an eaſy method of conſidering the ſeveral anſwers that would reſult from different Latitudes aſſumed in a geometrical way; for if the ſeveral Latitudes aſſumed, (ſuppoſing them increaſing or decreaſing by equal differences of a Minute of a Degree) denote equal Diſtances in a right Line, whilſt correſpondent Perpendiculars are erected the Lengths of the reſpective Elapſed-times, a Curve drawn through the uppermoſt Ends of thoſe Perpendiculars, will (in uſual Caſes) have but a ſmall Curvature; and therefore, for ſmall intermediate Diſtances, the arithmetical Means between two ſuch adjacent Perpendiculars, will coincide with the true Perpendicular, without ſenſible Error. Hence it follows, that in ſuch Caſes, the Part of the Curve ſo treated, nearly coincides with a right Line, and therefore may be proportioned as a ſimilar Triangle; and where there is a ſufficient Difference ariſing in the Elapſed-time on each Minute of Latitude, the Latitude inferred from the two correct Altitudes and the Elapſed-time, may be (with reſpect to a ſpherical Earth) to a ſmall part of a Minute of a Degree; but a leſs Accuracy will be ſufficient for many Improvements in Geography and Navigation.

Here might be introduced ſeveral other Methods whereby the Problem admits of a Solution in a different Manner from this or any other that hath been publiſhed, but whatever Method be applied it cannot determine the Latitude more exactly than it is poſſible to find it from the original Data.

Otherwiſe the Elapſed-time ariſing from an erroneous Aſſumption of the Latitude may be conſidered thus. Let *LGE* be a part of the Meridian of the Place of Obſervation, in which *c* is a Point in the Horizon under the elevated Pole, *E* the true Pole, *G* the true Point of the Equator, *JPN* the true Parallel of Declination of the ſame name with the Latitude, and *LQ* a Parallel of Declination of a different name, *N* the Point where the former of theſe Parallels cuts the Horizon, and *NP* the Aſcenſional Difference, whilſt *PJ* is 90 Degrees of that Parallel of Declination.

Moreover, let *e* be the Place of the Pole and *ce* the Latitude, when it is aſſumed leſs than the Truth by *Ee*, and draw the like correſpondent Parallels of Declination to this leſſer

leſſer aſſumed Latitude, *np* will be Aſſenſional Difference, which is manifeſtly leſs than *NP*. This ſuggeſts a Variety of eaſy Methods whereby the aſſumed Latitude may be applied for getting the true Latitude, by a Compariſon with Data depending on the Semidiurnal Arcs or Aſcenſional-differences. But with reſpect to the Elapſed-time, it is evident that when *e* coincides with *E*, then will *p* coincide with *P*. Conſequently, when *c E* (that is, the true Latitude) is aſſumed, and a computation is made by it, the true Polar-diſtance and the two true Co-altitudes, the Elapſed-time by Computation will agree with that by Obſervation, excepting the nicer Corrections. But if *c e* be aſſumed for the Latitude they will not agree. The like may be concluded for Latitudes aſſumed greater than the Truth, as is manifeſt by the Diagram itſelf.

Hence alſo it follows, that when one Latitude is aſſumed greater and the other leſs than the Truth, the Errors in the computed Elapſed-times will be of different kinds, one greater and the other leſs than the Truth, and therefore the true Latitude correſponding with the true Elapſed-time will fall between them, this makes caſe the firſt before deſcribed. But when both of the aſſumed Latitudes are either greater or leſs than the Truth, that of the two correſponding Elapſed-times which agrees neareſt with the meaſured Elapſed-time, will ſhew which of the two Latitudes is neareſt the Truth; and conſequently how the proportional Part is either to be added or ſubtracted, to get the true Latitude required.

Thus may this Method be demonſtrated to be true both in the Theory and Practice, without entering into a Conſideration of the various Caſes of ſpherical Triangles which in that Method of Solution embarras the Problem; for, in the preſent Method of Solution, the Aſtronomer or Navigator has nothing more to do than to compute the Elapſed-time from the Data with which he is provided; which being done and compared with the Elapſed-time truly meaſured, it will appear (from ſuch Circumſtances as have been before mentioned in this Tract) how the true Latitude is to be approached, according to the Accuracy of the Obſervations and the Properties of the Circles of the Sphere.

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☞ When the Altitude of the Sun's lower Limb has been taken, his Semidiameter added thereto gives the Altitude of the Centre. The Mean Semidiameter is 16m. and 3″, his greateſt Semidiameter but a little more than a quarter of a Minute more than the Mean, and his leaſt Semidiameter as much leſs than the Mean. The times of the Year when the Sun's Semidiameter meaſures at a Medium are near the Beginning of April and the 20th. of September; the greateſt Semidiameter the Beginning of January and the leaſt Semidiameter the Beginning of July; from which it will be eaſy to judge what the Semidiameter may be at other times of the Year within a few Seconds of a Degree.

# A New TABLE of REFRACTION.

As a Table of Refractions is of great Uſe in the Practice of this New Method of finding the Latitude; this Tract concludes with the following computed by me.

| Altitude. D. M. | Refraction. M. |
|---|---|
| 0. 7 | 33 |
| 0.12 | 32 |
| 0.18 | 31 |
| 0.24 | 30 |
| 0.30 | 29 |
| 0.36 | 28 |
| 0.42 | 27 |
| 0.49 | 26 |
| 0.56 | 25 |
| 1. 4 | 24 |
| 1.13 | 23 |
| 1.23 | 22 |
| 1.33 | 21 |
| 1.43 | 20 |
| 1.54 | 19 |
| 2. 7 | 18 |
| 2.19 | 17 |

The intermediate Refractions may be nearly found, by a Part proportional.

☞ This Table is applicable for taking out the Refraction, to the nearest Minute of a Degree, in a mean State of the Atmoſphere.

| Altitude. D. M. | Refraction. M. |
|---|---|
| 2.33 | 16 |
| 2.50 | 15 |
| 3. 7 | 14 |
| 3.27 | 13 |
| 3.50 | 12 |
| 4.20 | 11 |
| 4.50 | 10 |
| 5.20 | 9 |
| 6.15 | 8 |
| 7.15 | 7 |
| 8.35 | 6 |
| 10.15 | 5 |
| 13. 0 | 4 |
| 17. 0 | 3 |
| 24. 0 | 2 |
| 47. 0 | 1 |
| 90. 0 | 0 |

The intermediate Refractions may be nearly found, by a Part proportional.

In this Table, the firſt Column ſhews the Altitude of any of the Celeſtial Bodies, oppoſite to which in the ſecond Column is the Refraction to be ſubtracted from the obſerved Altitude to reduce it to the true Altitude, for each Minute of Refraction and the Altitude anſwering thereto, from the Horizon to the Zenith. Conſequently, by ſubtracting this Refraction and the Dip in Page 8. the true Altitude is found.

The Semi-diameter is additive, for Altitudes of the Sun's lower Limb, the Minutes and Seconds mentioned in Page 19.

FINIS.